



*Date:* March 24, 2020

*To:* CFD validation challenge participants

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*Subject:* An axisymmetric outer slip-wall boundary derived from transonic hump RANS simulations

A procedure has been developed at Sandia National Laboratories to determine the contour of an appropriate effective radius ( $R^*(x)$ ) to be used as an axisymmetric inviscid (slip-wall) boundary to facilitate CFD simulations of the transonic, shock-induced separation experiment as defined in Beresh *et al.* (2020) and Lynch *et al.* (2020). A RANS simulation of Sandia's Trisonic Wind Tunnel with the axisymmetric hump model installed (as defined in Lynch *et al.* (2020)) has been conducted and compared against collected experimental data. This RANS simulation, which resolves the tunnel wall boundary layers, is referred to herein as the "tunnel-resolved" simulation. A  $k-\epsilon$  turbulence model was used. The data from this simulation was used to derive the contour of  $R^*(x)$ . The goal of this procedure was to define a contour that would produce nearly the same mass flow rate, streamwise "free stream" Mach number distribution, wall surface pressure distribution along the model, and wall shear stress magnitudes along the model as were produced with the tunnel-resolved simulation.

The procedure for deriving  $R^*(x)$  is summarized here for those who may wish to repeat this procedure with their own code and preferred turbulence model.  $x$ -normal planes of data for relevant state variables were taken at a series of  $x$  locations down the length of the tunnel from the volumetric field data produced by the tunnel-resolving simulation. These were then integrated to determine the total integrated mass flow rate ( $\dot{m}$ ) at each plane. At locations upstream of any influence from the axisymmetric model, the tunnel centerline velocity ( $U_\infty$ ) and density ( $\rho_\infty$ ) were then chosen as the appropriate velocity and density values to define a uniform circular "plug-flow." By simply dividing  $\dot{m}$  by  $\rho_\infty U_\infty$ , the appropriate plug-flow circular area was determined, from which  $R^*$  was calculated. This procedure does not work at any  $x$  location where any influence from the model is present, because of both the occupation of the tunnel centerline by the model and flow non-uniformity introduced by the presence of the model. At those locations, profiles of the azimuthal average of the streamwise velocity were examined to identify the appropriate radial distance  $R_f(x)$  from the tunnel centerline at which to define an approximate "free stream" state,  $U_\infty$  and  $\rho_\infty$ . This was taken as the locus of points furthest from the model centerline where a local minimum in the absolute value of radial derivative of stream-wise velocity was observed. A uniform plug flow was assumed within the annular region between  $R_f$  and  $R^*$ , and the associated mass flow rate through the annulus was set equal to the mass flow rate computed from the tunnel-resolved simulation for the region  $r > R_f$ . The resulting  $R^*(x)$  can be found in the Appendix.

Simulations using the  $R^*(x)$  contour as a slip-wall boundary were then conducted and results have been compared to the tunnel-resolved simulation. The comparisons showed the following differences:

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## Streamwise Mach Number Distribution

When comparing streamwise traces of the Mach number defined along the line  $y = z = 3.6$  inches, a maximum relative difference of 0.33% was observed between the two simulations. This occurred upstream of the hump. At the reference location of  $x/c = 0$ , the two simulations showed a difference in Mach number of 0.0019 with both values landing within the target distribution (see Figure 5 of Lynch *et al.*, 2020).

## Model Wall Surface Pressure

The maximum observed difference in the wall pressures was  $< 500$  Pa and occurred near the point at which the flow reattached downstream of the hump. This difference equates to a relative difference of  $\approx 0.6\%$ .

## Model Wall Shear Stress Magnitude

The maximum difference in the wall shear stress magnitude was observed to be  $\approx 40$  Pa. This difference is located near the trip location on the model, and is not believed to be directly related to the use of the slip-wall boundary. It also occurred well upstream of the regions of interest of the validation challenge. The largest observed difference that is believed to be attributable to the use of the slip-wall boundary was  $\approx 8$  Pa, and occurred exactly in the corner where the hump meets the straight cylinder, within the recirculation region. Over all remaining locations along the model body, an average difference of  $\approx 0.67$  Pa was observed, while the average wall shear stress magnitude was  $> 90$  Pa. The two simulations show flow separation at the exact same streamwise point while reattachment occurred at points which differed by  $\approx 0.01$  inches.

## Caveats

Our results suggest that the equivalent inviscid circular outer boundary defined by  $R^*(x)$  provides an acceptable match for RANS prediction of mean surface quantities on the model. However, note that the validity of this approach for any simulation, using any particular turbulence model (RANS or otherwise), depends also on the accuracy of the RANS turbulence model used to predict the thickness of the tunnel wall boundary layer. We have only limited validation data for the tunnel wall boundary layer, so we cannot make statements about the absolute accuracy of  $R^*(x)$ . Participants may wish to define their own equivalent inviscid boundary, or otherwise verify the appropriateness of the present definition using their own numerical experiments.

## Bibliography

- Beresh, S. J., Barone, M. F., Dowding, K. J., Lynch, K. P., Miller, N. E., and Lance, B. W., "A CFD Validation Challenge for Transonic, Shock-Induced Separated Flow: Approach and Metrics," *AIAA Paper 2020-1308*, January 2020.
- Lynch, K. P. and Lance, B. W. and Lee, G. S. and Naughton, J. W. and Miller, N. E. and Barone, M. F. and Beresh, S. J. and Spillers, R. W. and Soehnel, M. M., "A CFD Validation Challenge for Transonic, Shock-Induced Separated Flow: Experimental Characterization," *AIAA Paper 2020-1309*, January 2020.

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## Appendix: $R^*$ values

The  $R^*(x)$  data. If the following data is copied and pasted into a *Rstar.dat* file, it can be imported into Pointwise directly as a database connector and revolved to create a domain for the slip wall. Alternatively, the *Rstar.dat* file will also be provided with this document.

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